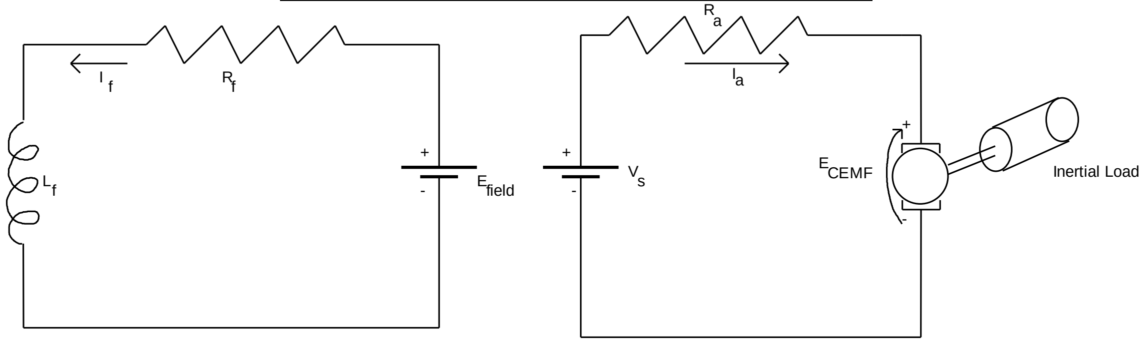
Dynamic Characteristics of a DC Motor



Separately excited field. Armature control

Angular acceleration $= \frac{d\omega}{dt}$ in Radians\second

Torque $= T$, $F =$ friction factor, $F\omega =$ friction proportion al to speed

$T = J\frac{d\omega}{dt} + F\omega$ but Torque for DC motor $T = K_f I_f I_a$

where K_f is a factor that is function of motor geometry

I_f is field current

I_a is armature current

sometimes we group $K_f I_f$ and call it K_T. This is done when the field current is held constant

$E_{CEMF} = K_g\omega$ Where K_g function of motor geometry

$$T = K_T I_a = J\frac{d\omega}{dt} + F\omega$$

$$I_a = \frac{V_S - K_g\omega}{R_a}$$

$$K_T[\frac{V_S - K_g\omega}{R_a}] = J\frac{d\omega}{dt} + F\omega$$

Rewrite $K_T[\frac{V_S - K_g\omega}{R_a}] = J\frac{d\omega}{dt} + F\omega$ into an expression like $\tau\frac{d\omega}{dt} + \omega = K_S V_S$ by putting like terms on each side

$$\frac{K_T}{R_a}V_S - \frac{K_T K_g}{R_a}\omega = J\frac{d\omega}{dt} + F\omega$$

$$\frac{K_T}{R_a}V_S = J\frac{d\omega}{dt} + \omega\left(\frac{K_T K_g}{R_a} + F\right)$$

$$\left(\frac{\frac{K_T}{R_a}}{\frac{K_T K_g + R_a F}{R_a}}\right)V_S = \left(\frac{J}{\frac{K_T K_g + R_a F}{R_a}}\right)\frac{d\omega}{dt} + \omega$$

$$\left(\frac{K_T}{K_T K_g + R_a F}\right)V_S = \left(\frac{R_a J}{K_T K_g + R_a F}\right)\frac{d\omega}{dt} + \omega$$

compare the above expression to: $\tau\frac{d\omega}{dt} + \omega = K_S V_S$

These are both first order Linear Differential Equation. Comparing coefficients,

$$\tau = \frac{J}{\frac{K_T K_g + R_a F}{R_a}}$$

$K_S = \dfrac{K_T}{K_T K_g + R_a F}$, K_S is referred to as the speed constant

$\tau\frac{d\omega}{dt} + \omega = K_S V_S$, units of K_S are $\dfrac{\text{Rads / sec}}{\text{volt}}$

use the following LaPlace transform pair to find the transfer function for the DC motor

$$\frac{d\omega(t)}{dt} \Leftrightarrow s\omega(s) \qquad \omega(t) \Leftrightarrow \omega(s)$$

$$K_S V_S(s) = \tau\, s\omega(s) + \omega(s)$$
$$= (\tau\, s + 1)\omega(s)$$

$\therefore \dfrac{\omega(s)}{V_S(s)} = \dfrac{K_S}{\tau\, s + 1}$ first order transfer function, V_s is supply voltage

Suppose $V_S(s) = 100$ V DC, LaPlace of constant supply voltage of 100 is $\dfrac{100}{s}$

Suppose $\omega(s) = \dfrac{100K_S}{s(\tau\, s + 1)}$

using LaPlace transform pair $\dfrac{K}{s(as+1)} \Leftrightarrow K(1 - e^{\frac{-t}{a}})$

then $\omega(t) = 100K_S(1 - e^{\frac{-t}{\tau}})$

Steady state speed is the final speed after the transient has disappeared. Theoretically it occurs when t is infinity. When this occurs, the term $e^{\frac{-t}{\tau}}$ goes to zero. Practically, after about 5τ the speed is about 99% of the steady state speed. The steady state speed is: $\omega_{\text{Steady State}} = 100K_S$ for $V_S = 100$. Another way of viewing this is to realize that at steady state the angular acceleration is 0.

$\dfrac{d\omega}{dt}$ goes to 0, that is

$\tau\dfrac{d\omega}{dt}$ goes to 0. The equation $K_S V_S = \tau\dfrac{d\omega}{dt} + \omega$ becomes

$\omega_{\text{Steady State}} = 100K_S$ for $V_S = 100$