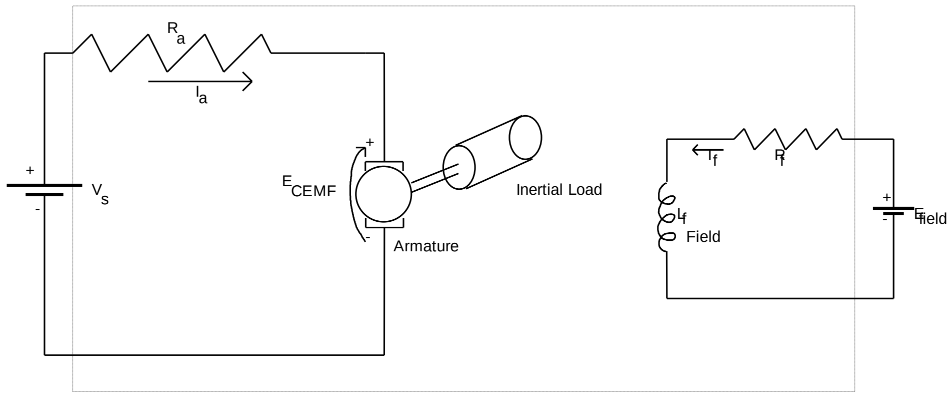
DC Motor Position Control

Remember the LaPlace transform of the DC motor shown below is:

$$\frac{\omega(s)}{V_S(s)} = \frac{K_S}{\tau s + 1}$$ first order transfer function , V_s is supply voltage

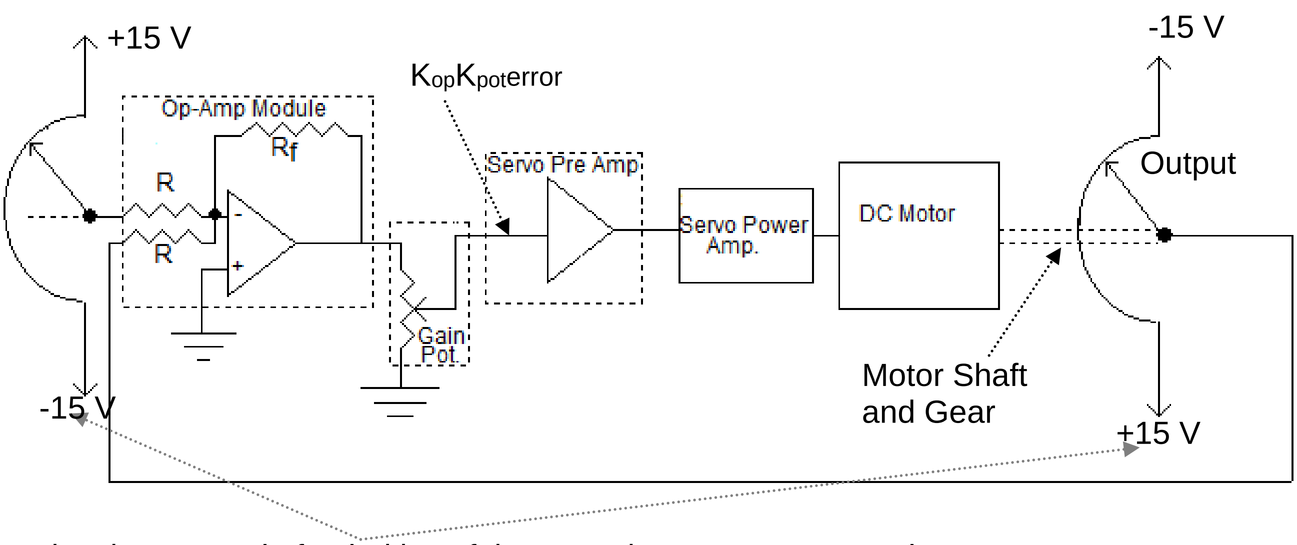


DC MOTOR

See your previous notes for the development of the transfer function.

Shown below is a position control servo system. The object of this control system is to position the output potentiometer from the input or setpoint potentiometer. These types of control systems are used for aircraft controls, XY table position controls, robots etc. When the angular postions of the potentiometers are different a voltage other than 0 will exist on the output of the op amp module forcing the motor to turn in the direction to reduce the error.

When the input and output positions are equal, the output from the op amp module will be 0 and the motor will stop.

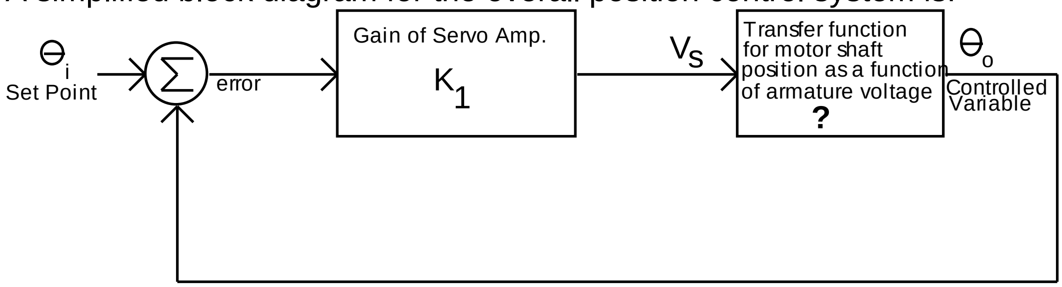


Notice the reversal of polarities of the potentiometer power supply

Θ_i is the reference or target position or set point

Θ_O is the output or controlled variable position

A simplified block diagram for the overall position control system is:



We know that the transfer function for the DC motor angular velocity vs supply voltage is:

$$\frac{\omega(s)}{V_S(s)} = \frac{K_S}{\tau s + 1}$$

To find out the transfer function for the DC motor shaft position vs supply voltage we have to look at the relationship between angular velocity and position. That is:

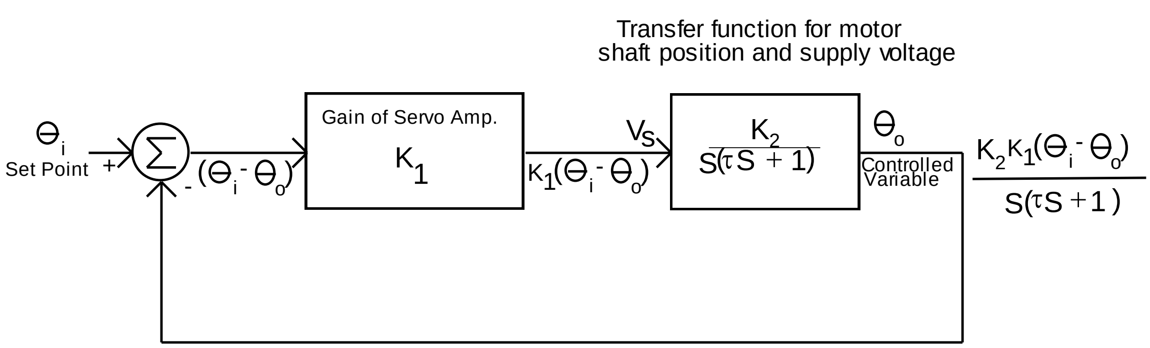
$$\omega = \frac{d\theta}{dt}$$

using the LaPlace relationship $\omega(s) = S\theta(s)$

$$\frac{\omega(s)}{V_s(s)} = \frac{K}{\tau S + 1}$$

then $$\frac{S\theta(s)}{V_s(s)} = \frac{K}{\tau S + 1}$$

or $$\frac{\theta(s)}{V_s(s)} = \frac{K}{S(\tau S + 1)}$$



$$\theta_o = \frac{K_1K_2(\theta_i - \theta_o)}{s(\tau s + 1)}$$

$$\theta_o + \frac{K_1K_2\theta_o}{s(\tau s + 1)} = \frac{K_1K_2\theta_i}{s(\tau s + 1)}$$

$$\theta_o(1 + \frac{K_1K_2}{s(\tau s + 1)}) = \frac{K_1K_2\theta_i}{s(\tau s + 1)}$$

$$\frac{\theta_o}{\theta_i} = \frac{\frac{K_1K_2}{s(\tau s + 1)}}{(1 + \frac{K_1K_2}{s(\tau s + 1)})}$$

$$\frac{\theta_o}{\theta_i} = \frac{K_1K_2}{s(\tau s + 1) + K_1K_2}$$

$$\frac{\theta_o}{\theta_i} = \frac{1}{\frac{\tau s^2}{K_1K_2} + \frac{s}{K_1K_2} + 1}$$

If you compare this transfer function to the general form of the 2'nd order transfer function:

$$\frac{1}{(\frac{1}{\omega_n^2})S^2 + 2(\frac{\rho}{\omega_n})S + 1}$$

ω_n is the ***undamped*** natural frequency

ρ is the damping coefficient

comparing coefficients of the 2 transfer functions, we can say:

$$\frac{1}{(\frac{1}{\omega_n^2})S^2 + 2(\frac{\rho}{\omega_n})S + 1}$$

$$\frac{1}{\omega_n^2} = \frac{\tau}{K_1K_2}$$

or $\omega_n = \sqrt{\frac{K_1K_2}{\tau}}$

also $2(\frac{\rho}{\omega_n}) = \frac{1}{K_1K_2}$ or $\rho = \frac{\omega_n}{2K_1K_2}$ then

$$\rho = \frac{\sqrt{\frac{K_1K_2}{\tau}}}{2K_1K_2} = \frac{1}{2\sqrt{\tau K_1K_2}}$$

When the gain of the servo amplifier K_1 increases then the damping coefficient ρ will decrease tending to make the step response of the position control system more oscillatory.

Increasing the servo amp gain K_1 also causes the undamped natural frequency ω_n to increase.

Remember however, that if the step response is underdamped, the period of the decaying oscillation is actually less than the undamped natural frequency ω_n. It is in fact $\omega_n\sqrt{1-\rho^2}$ and is referred to as the damped frequency ω_d.

For the general form of the 2'nd order transfer function $\dfrac{1}{(\frac{1}{\omega_n^2})S^2 + 2(\frac{\rho}{\omega_n})S + 1}$, the underdamped step response can be expressed as:

$$\text{step response} = 1 - \left(\frac{e^{-\rho\omega_n t}}{\sqrt{1-\rho^2}}\right)\sin(\omega_n(\sqrt{1-\rho^2})t + \varphi)$$

$$\text{where } \varphi = \tan^{-1}\frac{\sqrt{1-\rho^2}}{\rho}$$