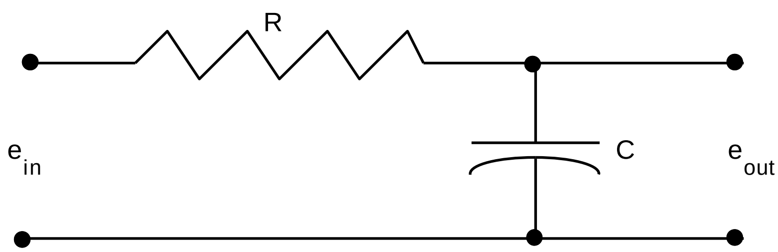
Frequency Response in Process Control Systems

- Relates Gain of a transfer function and the phase shift which occurs when a sinusoidal input is varied over a wide range of frequencies.
- Can be determined experimentally or by analysis.
- Another way to determine the characteristics of a transfer function.

$$\text{Gain } G = \frac{\text{Output Amplitude}}{\text{Input Amplitude}}$$

- in decibals, Gain = $20\log_{10}G$, phase shift ϕ expressed in degrees



$$\tau\frac{de_{out}}{dt} + e_{out} = e_{in}$$

$$\frac{E_o}{E_i} = \frac{1}{\tau s + 1}$$

$$\text{where } \tau = RC$$

suppose $\tau = RC$ = 0.1 sec then for a sinusoidal input, the gain of the transfer function can be calculated at a given radian frequency by substituting $j\omega$ for s in the above transfer function and then convert the expression to polar form. That is:

$$\frac{E_o}{E_i} = \frac{1}{0.1j\omega + 1}$$

Remember that for a + jb, magnitude is $\sqrt{a^2 + b^2}$ and phase angle is $\tan^{-1}\frac{b}{a}$

For ω=10, the expression becomes:

$$\frac{E_o}{E_i} = \frac{1}{j1 + 1}$$

$$= \frac{1}{\sqrt{1^2 + 1^2}\angle\tan^{-1}1}$$

$$= 0.707\angle - 45°$$

For ω=20, the expression becomes:

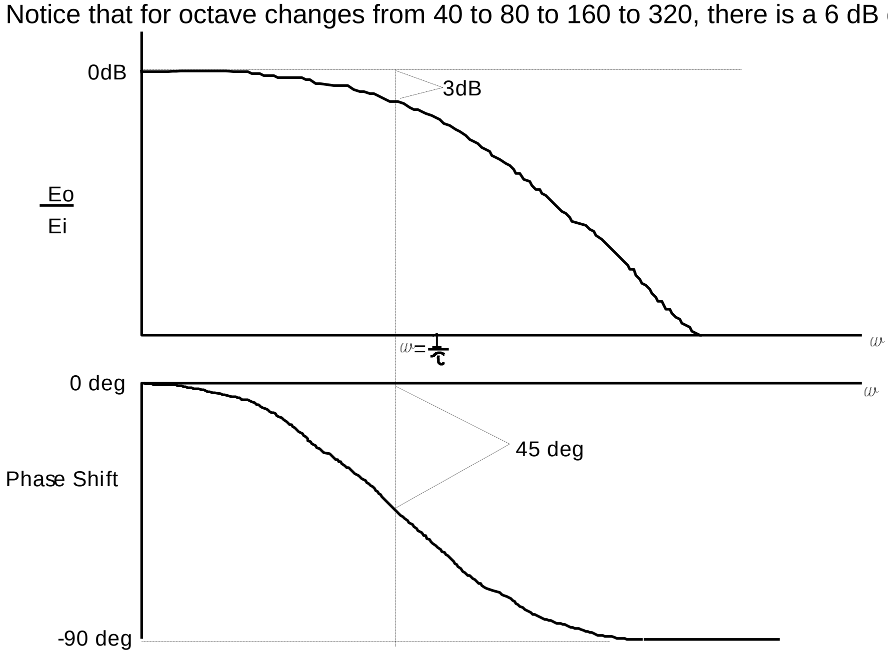
$$\frac{E_o}{E_i} = \frac{1}{j2+1}$$

$$= \frac{1}{\sqrt{2^2+1^2}\angle \tan^{-1} 2}$$

$$= 0.44\angle -63.4^\circ$$

ω	$\left\|\frac{E_0}{E_1}\right\|$	ϕ (deg)	$20\log_{10}\left\|\frac{E_0}{E_1}\right\|$ dB
5	0.89	-26.6	-1.0
10	0.71	-45	-3.0
20	0.44	-63.4	-7.1
30	0.31	-71.6	-10.2
40	0.24	-76	-12.4
80	0.12	-82.8	-18.1
160	0.06	-86.4	-24.1
320	0.03	-88.2	-30.1

Notice that for octave changes from 40 to 80 to 160 to 320, there is a 6 dB drop



Frequency Response of RC Circuit (a first order lag)

Note that at $\omega = \frac{1}{\tau}$ gain is down by 3 dB and the phase shift is -45 deg.

Some Rules of Transfer Functions

1) Overall transfer function of several components in series is equal to the product of the transfer functions of the individual components.

2) Overall gain of several components in series is equal to the product of the gain of the individual components.

3) Overall phase angle of several components in series is equal to the sum of the phase angles of individual components.

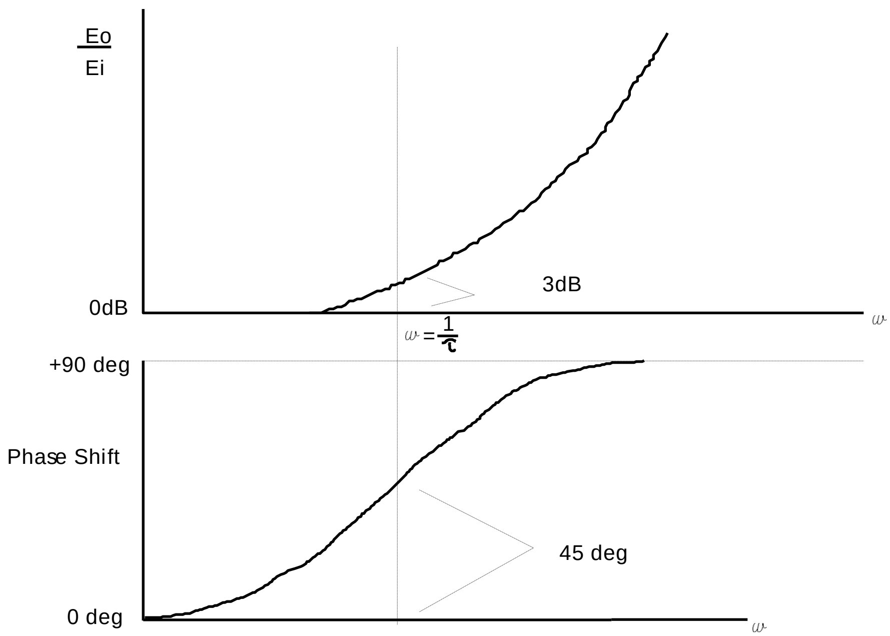
4) Overall frequency response of several components can be determined on a Bode diagram graphically by adding the decibel gains and adding the phase angles of individual components.

Frequency Response of First Order Lead

$$\frac{E_o}{E_i} = \tau s + 1$$

$$\text{Mag} = \sqrt{\omega^2 \tau^2 + 1}$$

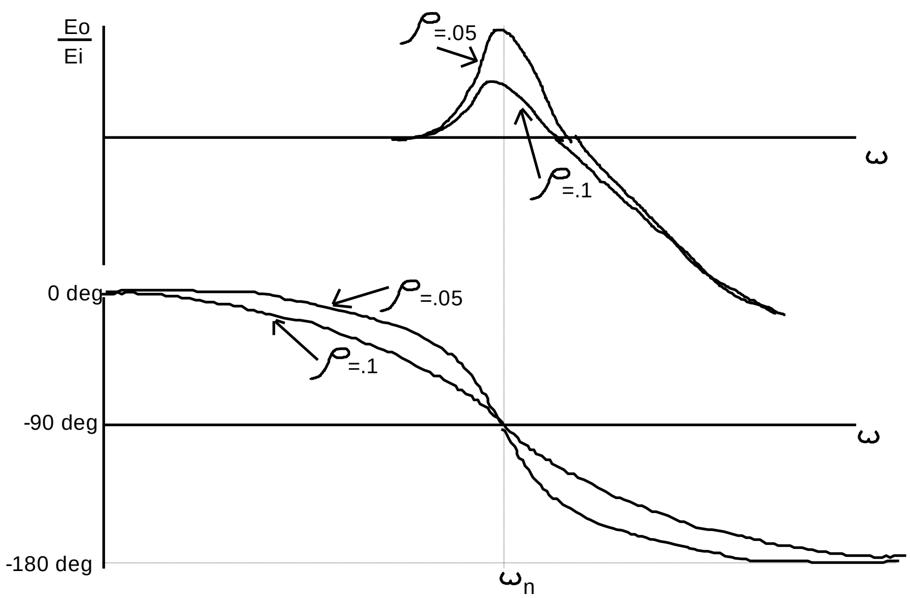
$$\phi = \tan^{-1} \frac{\omega\tau}{1}$$



Frequency Response of 2'nd Order System

$$\frac{E_o}{E_i} = \frac{1}{\frac{1}{\omega_n^2}s^2 + \frac{2\rho}{\omega_n}s + 1}$$

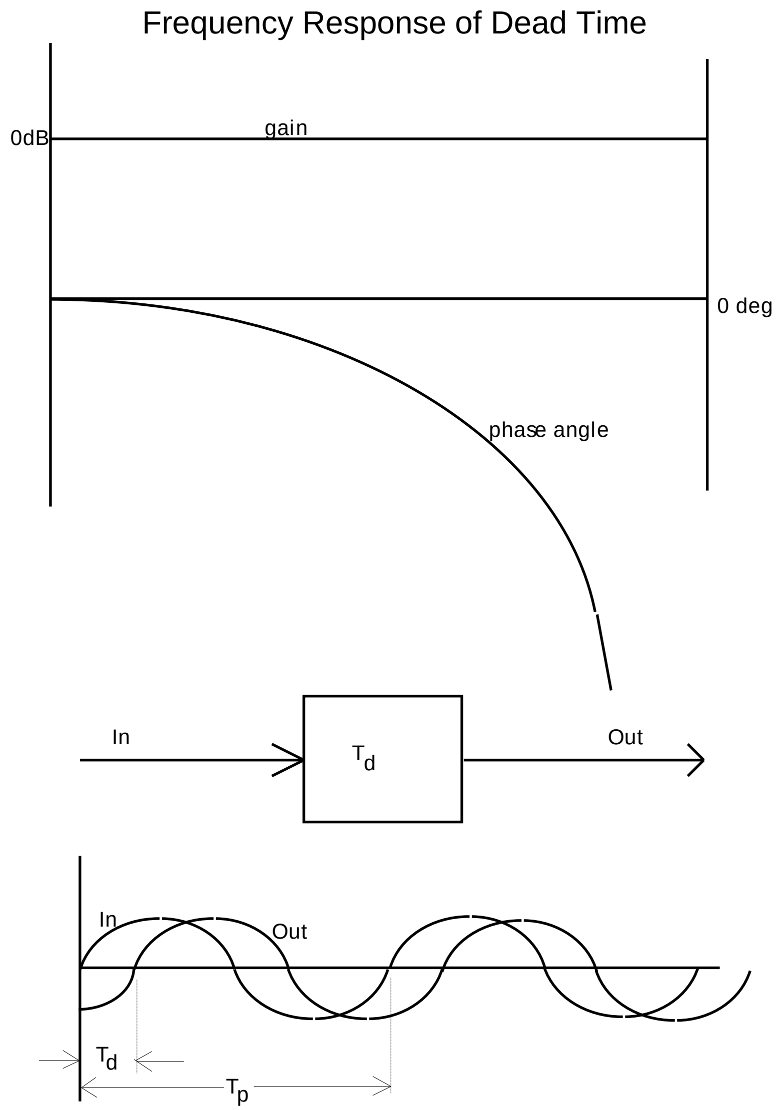
On the sketch on the next page, notice that near $\omega = \omega_n$, the natural frequency of the system, the gain is greater than 0 dB for the values of damping factor shown (0.05 and 0.1). That is , if the value of the damping factor less than 1.0, the gain will actually exceed 0 dB for a frequency near the natural frequency. However damping factors greater than 1, the gain will steadily decrease from 0 dB.



Frequency Response of Pure Dead Time

For pure delay or dead time with no attenuation, transfer function is:

$$\frac{E_o}{E_i} = e^{-T_d s}$$

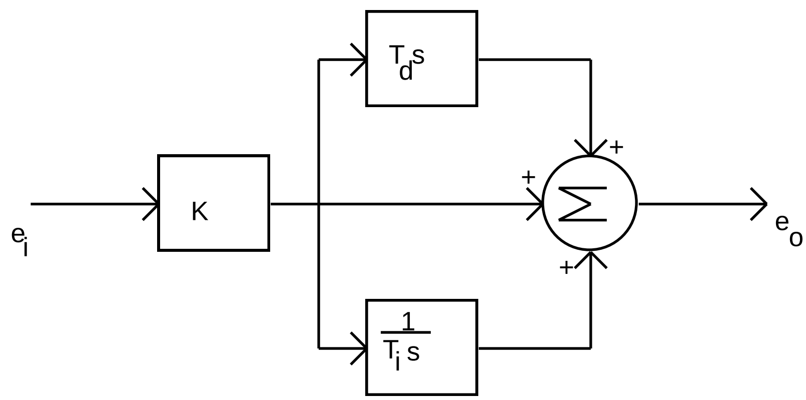


If T_d is dead time delay and T_p is period of sine wave, then the delay T_d represents a phase angle of :$\phi_{T_d} = \frac{T_d}{T_p} \times 2\pi$ rads. Since $f = \frac{1}{T_p}$ then $\phi_{T_d} = T_d \times 2\pi f$ or $\phi_{T_d} = T_d \times \omega$ rads.

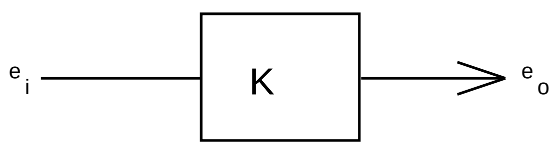
From these expressions you can see that as the frequency increases, the phase shift increases. The phase shift never stops increasing. This is why dead time can significantly contribute to the instability of a loop.

Frequency Response of the PID Components

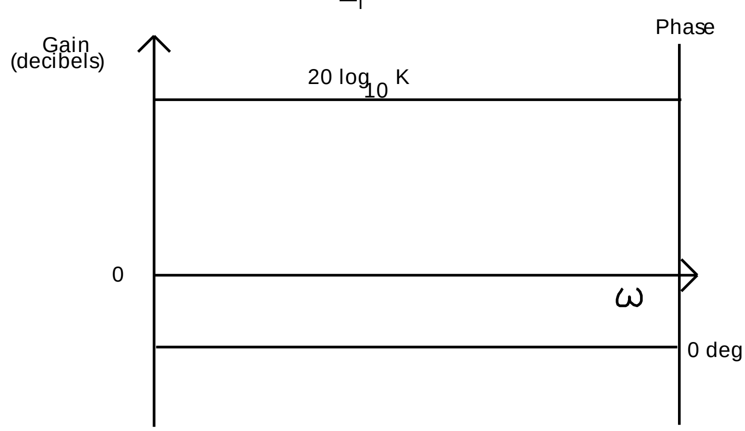
PID Controller



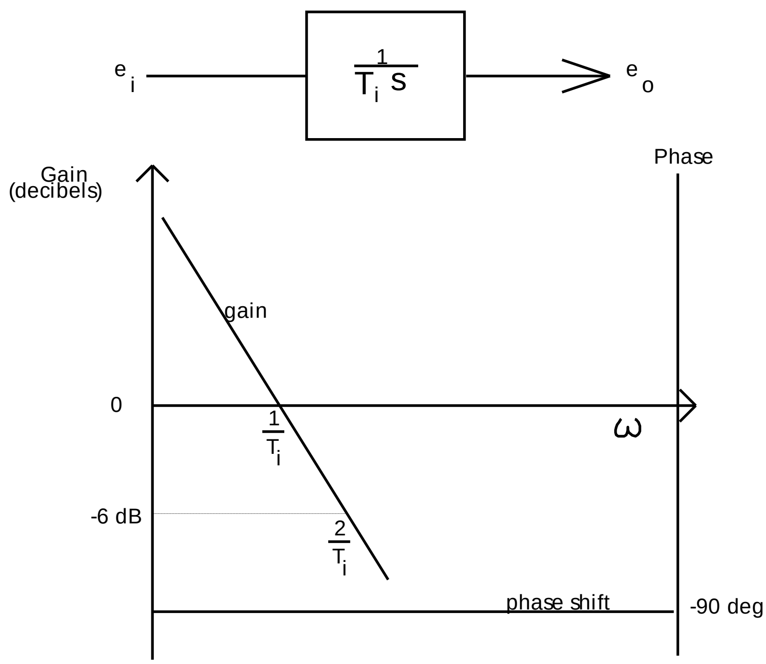
Frequency Response of Proportional



$$\frac{E_o}{E_i} = K$$



Frequency Response of Integrator



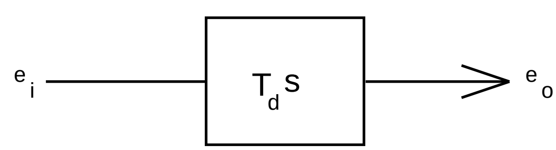
$$\frac{E_o}{E_i} = \frac{1}{T_i s}$$

substituting $j\omega$ for s

$$\text{Gain} = \frac{1}{T_i j\omega} = \left|\frac{1}{T_i\omega}\right| \angle -90^\circ$$

Notice that gain decreases as frequency increases. Gain very high at low frequencies.

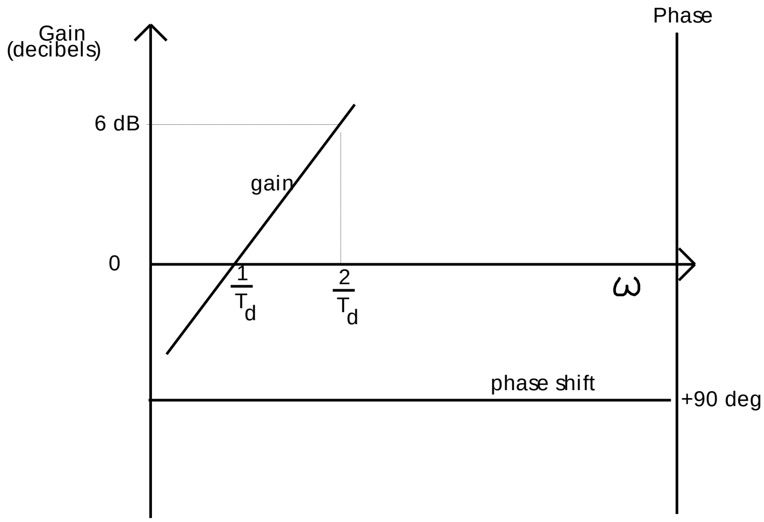
Frequency Response of Derivative



$$\frac{E_o}{E_i} = T_d s$$

substituting $j\omega$ for s

$$\text{Gain} = T_d j\omega = \left|T_d\omega\right| \angle 90^\circ$$



Notice that phase shift is +90 deg and gain increases with increasing frequency.