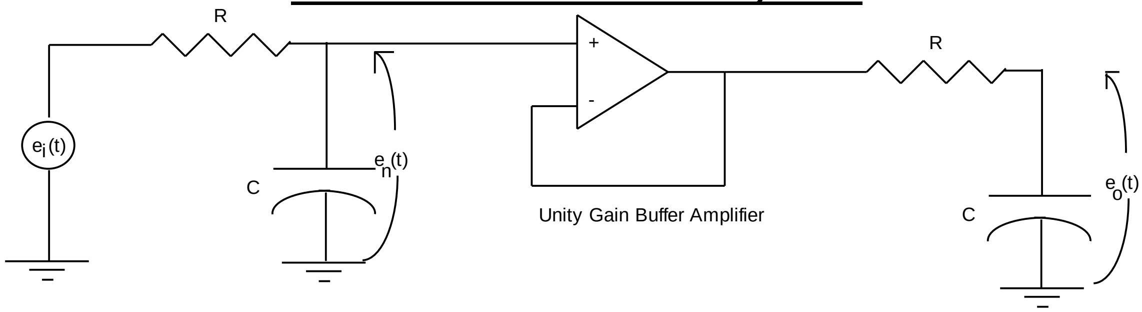
Second Order Electronic RC Systems



- Non Interacting - buffer amplifier prevents 2'nd stage from loading down first stage
- Buffered
- Independent of each other

$$\tau \frac{de_n}{dt} + e_n = e_i \quad \text{(eqn \#1)}$$

$$\tau \frac{de_o}{dt} + e_o = e_n \quad \text{(eqn \#2)}$$

$$\tau \frac{d(\tau \frac{de_o}{dt} + e_o)}{dt} + (\tau \frac{de_o}{dt} + e_o) = e_i$$

$$\tau^2 \frac{d^2 e_o}{dt^2} + \tau \frac{de_o}{dt} + \tau \frac{de_o}{dt} + e_o = e_i$$

$$\tau^2 \frac{d^2 e_o}{dt^2} + 2\tau \frac{de_o}{dt} + e_o = e_i$$

- The above is a second order differential equation because of second derivative term.

$$\tau^2 \frac{d^2 e_o}{dt^2} + 2\tau \frac{de_o}{dt} + e_o = e_i$$

Note the following LaPlace Transform pairs

$$\frac{d^2 e}{dt^2} \Leftrightarrow s^2 E(s)$$

$$\frac{de}{dt} \Leftrightarrow s^1 E(s)$$

$$e \Leftrightarrow s^0 E(s) \text{ or } e \Leftrightarrow E(s)$$

$$K \Leftrightarrow s^{-1} K \text{ or } K \Leftrightarrow \frac{K}{s}$$, where K is not a function of time

then

$$\tau^2 s^2 E_o(s) + 2\tau\, sE(s) + E_o(s) = E_i(s)$$

FIND: $\frac{E_o(s)}{E_i(s)}$?

$$E_o(s)[\tau^2 s^2 + 2\tau s + 1] = E_i(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{\tau^2 s^2 + 2\tau s + 1}$$

$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{1}{(\tau s + 1)(\tau s + 1)}$$