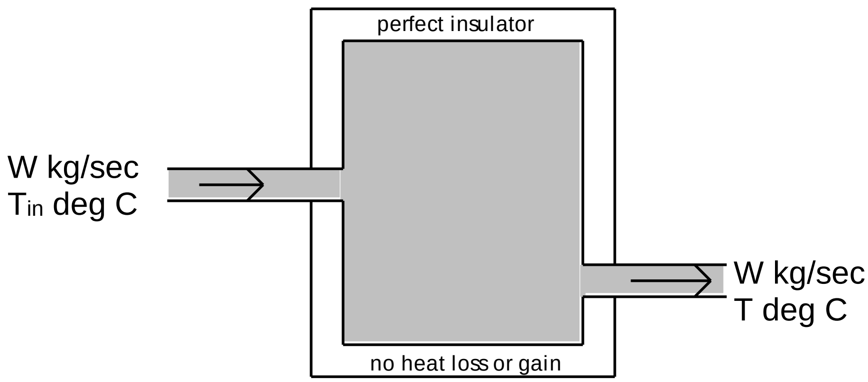
A SingleTime Constant Thermodynamic Process

* C_p - Heat capacity of liquid (J/kg/deg C)
* D - Density of liquid (kg/m^3)
* V - Volume held up in tank (m^3)



* Assume no heat losses from tank (perfectly insulated)

* Energy Balance, **Heat In** - **Heat Out** = **Heat Accumulated**

$$\text{Heat In } \left(\frac{J}{sec}\right) = WC_pT_{in} \quad \left(\frac{kg}{sec} \cdot \frac{J}{kg.^oC} \cdot {}^oC\right)$$

$$\text{Heat Out } \left(\frac{J}{sec}\right) = WC_pT \quad \left(\frac{kg}{sec} \cdot \frac{J}{kg.^oC} \cdot {}^oC\right)$$

$$\text{Heat Accum. } \left(\frac{J}{sec}\right) = VDC_p \frac{dT}{dt} \quad \left(m^3 \cdot \frac{kg}{m^3} \cdot \frac{J}{kg.^oC} \cdot \frac{^oC}{sec}\right)$$

Then

$$WC_pT_{in} - WC_pT = VDC_p \frac{dT}{dt} \quad \text{or}$$

$$VDC_p \frac{dT}{dt} + WC_pT = WC_pT_{in}$$

$$\frac{VD}{W} \frac{dT}{dt} + T = T_{in}$$

$\frac{VD}{W}$ is time constant τ, check units

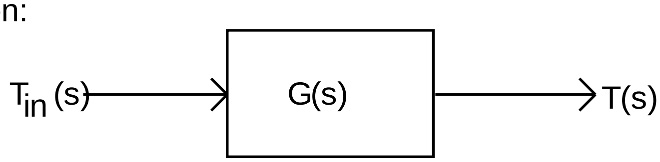
$$\left[\frac{m^3 \cdot \frac{kg}{m^3}}{\frac{kg}{sec}}\right] \Rightarrow sec$$

$$\tau \frac{dT}{dt} + T = T_{in} \quad \text{(first order differential equation)}$$

Solution

Can be solved by LaPlace transform technique.

Find transfer function:



$$\tau \frac{dT}{dt} + T = T_{in}$$

can be rewritten as

$\tau s T(s) + T(s) = T_{in}(s)$ or

$$\frac{T(s)}{T_{in}(s)} = \frac{1}{\tau s + 1}$$ transfer function for first - order process

How will temperature behave if T_{in} is suddenly (**step change**) increased ?

Use LaPlace:

$$T(s) = \left(\frac{1}{\tau s + 1}\right) T_{in}(s)$$

a **step change** for $T_{in}(s)$ is represented by T_{in}/s where T_{in} represents the magnitude of the **step change** in deg C. Then

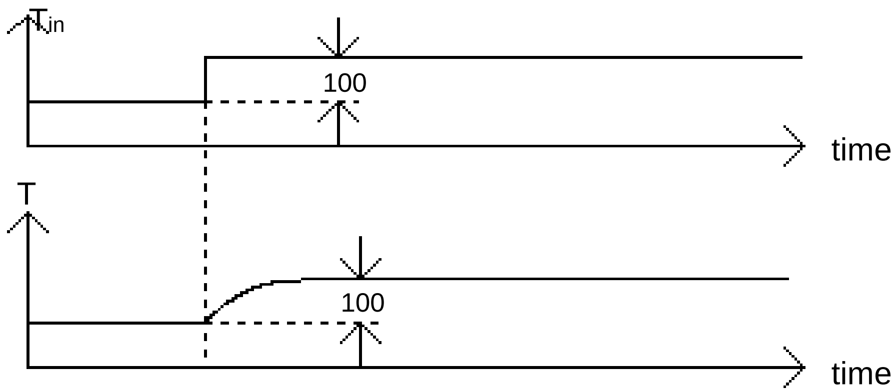
$$T(s) = \left(\frac{1}{\tau s + 1}\right) \frac{T_{in}}{s}$$

From a table of LaPlace transforms the <u>Inverse</u> LaPlace transform is taken of T(s) converting the expression to the time domain. Thus

$$T = T_{in}\left(1 - e^{-\frac{t}{\tau}}\right)$$

If the initial temperatures T and T_{in} were 0 deg C, T would represent temperature as a function of time for a step change in T_{in} from 0 deg C.

If the initial temperatures were say 100 deg C, then T would represent the change in temperature from an initial temperature of 100 deg C. Also T_{in} would be the step change in temperature from the initial temperature.



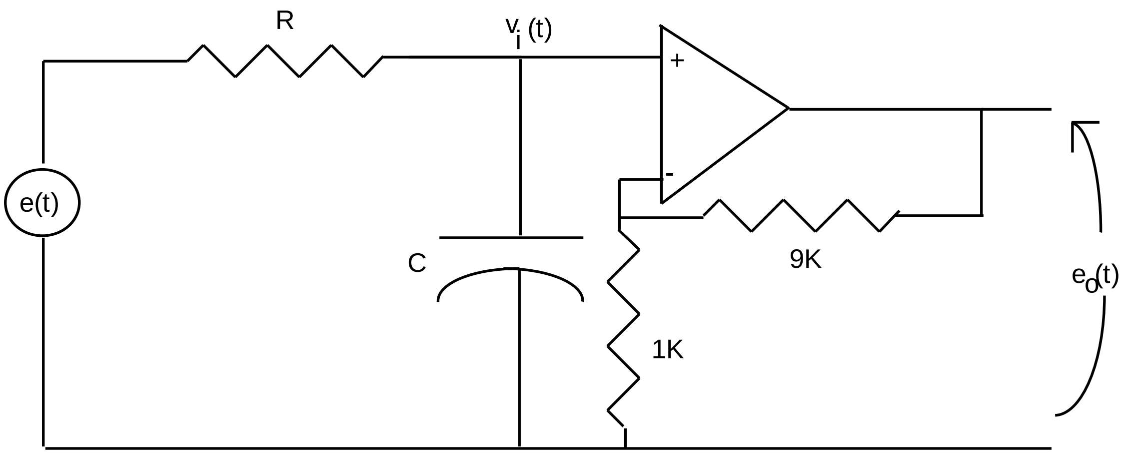
Notice that the steady state temperature is 100 deg C. i.e. a 100 C deg change results in a 100 C deg change in output temperature. The process gain is T/ T_{in} or 100/100 or 1 (as can be seen from the transfer function).

Notice the **similarity** below in the mathematics for the **thermal** and the **electronic** system.

$$\tau \frac{de_o}{dt} + e_o = e_i \Rightarrow \frac{E_o}{E_i} = \frac{1}{\tau s + 1}$$

$$\tau \frac{dT}{dt} + T = T_{IN} \Rightarrow \frac{T}{T_{in}} = \frac{1}{\tau s + 1}$$

For the circuit below, what is the steady state value of the output voltage e_o if e_i is a 1volt DC source.



$$\begin{aligned} e_o &= v_i(t) + \frac{v_i(t)}{1k\Omega} \times 9k\Omega \\ &= v_i(t) + 9v_i(t) \\ &= 10v_i(t) \end{aligned}$$

but $v_i(t)$ is the voltage across the capacitor. In the steady state, it will equal the DC source voltage of 1 volt. Then the steady state output voltage will be 10 volts. The amplifier part of the circuit just multiplies the voltage across the capacitor. The ***steady state gain*** of this circuit is 10. In controlsystems terminology, this gain is referred to as the ***Static or Process or DC Gain.***

Suppose that $e_i(t) = 2V$, the transient voltage across the capacitor could be found using LaPlace. That is:

$$E_o(s) = \left[\frac{2}{s}\right]\left[\frac{10}{\tau s+1}\right]$$
$$= \frac{2}{s} \times \frac{10}{\tau s+1}$$
$$= \frac{20}{s(\tau s+1)}$$
$$e_o = 20(1 - e^{-t/\tau})$$

This equation characterizes the **process dynamics** of the **first order** electronic RC process with a DC gain of 10

The plot for the circuit would look like that shown below