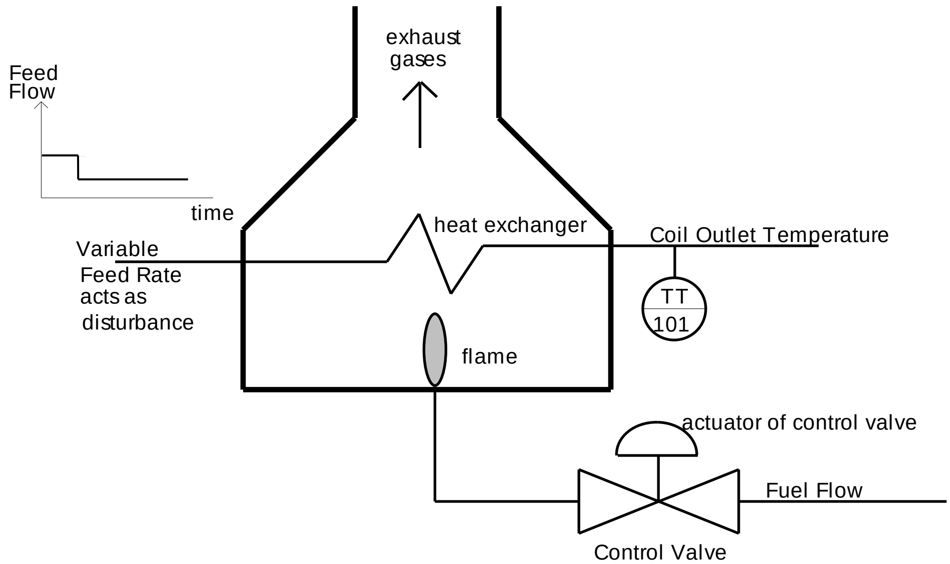
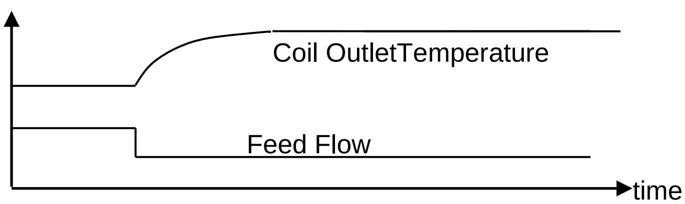
Process Dynamics

Process Dynamics mathematically describes the behavior of the controlled variable response with respect to time.

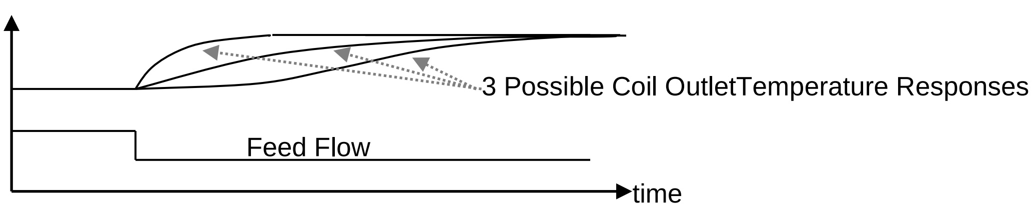
No Feedback Control – Open Loop





The above graph shows the coil outlet temperature rising due to the step decrease in feed flow. The manner in which the temperature changes with respect to time for a given change in input, in this case a step change, is referred to as the process dynamics of the system. Mechanical, chemical. thermodynamic, and electromechanical systems have characterisitic system dynamics that can be approximately analyzed mathematically. The electronic RC "process" will first be analyzed to investigate its time response to a step change in voltage.

Depending on the physical characteristics of the furnace and the properties of the fuel and feed the response could look like any of those shown below:



Electronic RC Circuit “Process”

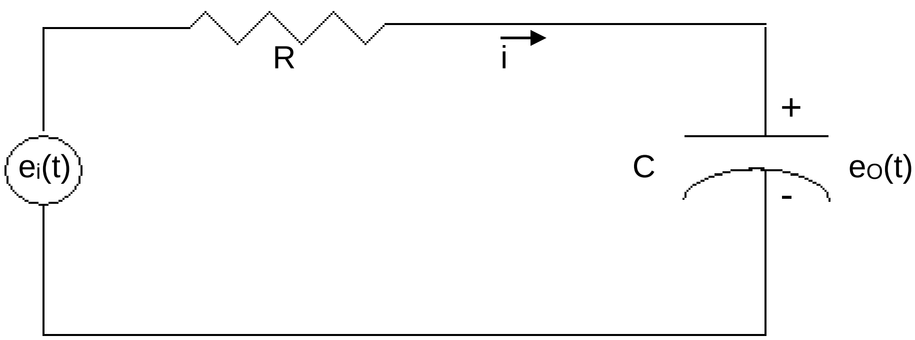
For a Capacitor, $v_C \propto q = \frac{1}{C}q$

$$\frac{dv_C}{dt} = \frac{1}{C}\frac{dq}{dt}$$

$$\frac{dq}{dt} = i \text{ , } v_C = \frac{q}{C}$$

$$\frac{dv_C}{dt} = \frac{1}{c}i_c$$

Therefore, $i_c = C\frac{dv_C}{dt}$



Then for the RC circuit above, $v_C = e_O(t)$, e is a function of time

$$i = C\frac{de_o}{dt} = \frac{e_i - e_o}{R}$$

$$RC\frac{de_o}{dt} = e_i - e_o$$

$$RC\frac{de_o}{dt} + e_o = e_i$$

$$RC = \tau$$

$$\tau\frac{de_o}{dt} + e_o = e_i$$

This is a first order differential equation. It can be solved in a couple of ways. We will use the **LaPlace** transform to solve it for a step input which is the input when $e_i(t)$ is a battery.

The **Laplace** transform is a technique for circuit analysis that is used for the calculation of a systems time response.

When a certain input signal such as a step change provided by a battery is applied to the system the output as a function of time can be calculated using **Laplace**.

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The **Laplace** transform converts the input signal and the differential equation describing the system, into algebraic epressions in what is known as the complex frequency domain. The output in the complex frequency domain is the product of the input **LaPlace** transform and the system **LaPlace** transform. The output of the system as a function of time is the **Inverse Laplace** transform of this product.

LAPLACE:

	Time domain	transforms to/from	Frequency Domain
1	$e_o(t)$ (function of time)	$\Longleftrightarrow$	$E_o(s)$
2	$\frac{de_o(t)}{dt}$ (derivative of a function of time)	$\Longleftrightarrow$	$sE_o(s)$
3	K (constant)	$\Longleftrightarrow$	$\frac{K}{s}$
4	$K(1-e^{-t/a})$	$\Longleftrightarrow$	$\frac{K}{s(as+1)}$

For the differential equation for the RC circuit.

$$\tau \frac{de_o}{dt} + e_o = e_i$$

$$E_o(s)[\tau s + 1] = E_i(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{\tau s+1}$$

This is called a transfer function and represents a *first order* or a *single pole low pass filter*
If $E_i(s)$ is a battery (which is a constant) of voltage V_{bat}, then $E_i(s)$ can be replaced by

$$\frac{V_{bat}}{S}$$

$$\text{Then } E_o(s) = \frac{V_{bat}}{s}\frac{1}{\tau s+1}$$

Then using Laplace transform 4 from the above table:

$$e_o = V_{bat}(1-e^{-t/\tau})$$

$$\therefore e_o = V_{bat}(1-e^{-t/RC})$$

This is the response (output voltage) of a first order system (one resistor and one capacitor) for a **STEP** input (DC battery).

With respect to an AC voltage source, LaPlace transforms can be used (but not proven here) to show that:

1) For a one capacitor, one resistor RC circuit with a sine wave input with variable frequency, the output amplitude of the sine wave will decrease or "roll off" at 20 dB per decade (6 dB per decade)

2) For a circuit with 2 resistors and 2 capacitors the output will decrease or "roll off" at 40 dB perdecade (12 dB per octave).

3) Similarly three RC's will decrease or "roll off" at 60 dB per decade (18 dB per octave)